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KINEMATIC ANALYSIS OF THREE DEGREE OF FREEDOM CLOSED CHAIN MECHANISM

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ABSTRACT

Mechanisms can be classified into two main categories based on their structures, namely, serial and parallel. Parallel structures are an alternative to serial ones, giving the important mechanical and kinematic advantages offered i.e. higher structural rigidity, higher motion accuracy, higher pay load capacity, low inertia, higher velocity and acceleration of the end effector. With increasing use of parallel structures, comes a need to develop a methodology for analysis of different parallel robot design.

For the motion analysis of any mechanism, initially the kinematic analysis is done. This paper presents a simple geometric approach to analyze, a novel unsymmetrical eight bar planar parallel mechanism with three degree of freedom. It consists of a closed chain five bar planer parallel mechanism, to which a three link serial structure is joined, forming a another closed chain mechanism. Hence, from the geometry of the mechanism, it contains three actuators, actuating each input link. In kinematic analysis - the direct kinematic analysis is carried out by decomposing the closed chain of the mechanism in different sub chains and selecting the origin. In the same way, the inverse kinematic analysis is done to calculate the joint values for a given end effector positions.

The method has been tested for the sample case to analyze the position of the end effector for different inputs by changing the origin of the mechanism. The analysis shows that, there is no effect on end effector position, though the origin of the mechanism gets change.

Keywords: *Closed chain parallel linkage mechanism, three degree of freedom mechanism, kinematic analysis, forward kinematic analysis, inverse kinematic analysis.*

NOMENCLATURE

C	Cos
S	Sin
L_1	Length of link 1
L_2	Length of link 2
L_3	Length of link 3
L_4	Length of link 4
L_5	Length of link 5
X_p	X-coordinate of point P
Y_p	Y-coordinate of point P
Θ	Angle of link 1 with ground
Θ_1	Input angle (angle between link 1&2)
Θ_2	Output angle(angle made by link 3 with horizontal)
Θ_3	Output angle(angle made by link 4 with horizontal)
Θ_4	Input angle(angle between link 1&5)
Θ_5	Input angle(angle between link 6&7)
Θ_6	Output angle(angle made by link 8 with horizontal)
Θ_7	Output angle(angle made by link 9 with horizontal)

INTRODUCTION

Mechanisms can be classified into two main categories based on their structures, namely, serial and parallel. The serial structure is one in which the links and joints are arranged in alternate fashion, with the constraint that closed loops are not formed, whereas a parallel structures has a closed loop consists of several kinematic chains connecting the fixed base to the end-effector. Theses mechanisms are widely used for robot manipulators. Serial robot manipulator, having mechanisms with serial structures, have a simpler structure, wider reachable area and having relatively simpler kinematics. Due to these advantages, serial mechanisms are extensively used in the industries, for assembling, welding, painting, etc. However, the serial structure leads to low rigidity, smaller load capacity, low stiffness and cannot reach high dynamic performances. Therefore they are not suitable for some applications where large load or high speed and accuracy are needed [1]. Parallel structures are an alternative to serial ones giving the important mechanical and kinematic advantages offered, i.e. high structural rigidity, highmotion accuracy, high payload capacity, low inertia, higher velocity and acceleration of the end effector. However, in contrast of these advantages parallel structures have a main drawback of small reachable area and a complex

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mathematical model as compared to serial structures. Hence, parallel robot manipulators are used in industry forprecise positioning and alignment [2]. With their increasing usecomes a need to analyze the parallel linkage mechanisms.

For the motion analysis of any mechanism, initially the kinematic analysis is done. The kinematic analysis of a parallel linkage mechanism is more difficult and complex as compared to serial linkage mechanisms. In kinematic analysis, two sorts of position analysis can be distinguished, namely Direct kinematic and inverse kinematic. The direct kinematic refers to the calculation of end effector position, orientation, velocity and acceleration, when the corresponding joint values are known. Inverse kinematics are used to calculate the joint values for a given end effector positions. This helps in formulating the algorithm required for controlling the robot. Normally, parallel manipulators present multiple direct kinematic and inverse kinematic solutions. There are many techniques proposed to solve the forward and inverse kinematic problems [3].

In the literatures, many three degree of freedom (DOF) planer parallel manipulators have been proposed. They are classified into two categories, namely, redundant and non-redundant based on the number of actuators.A redundant three DOF parallel manipulators has more than three actuators, whereas a non-redundant has only three actuators. Almost all the manipulators studied in the literatures are of symmetrical in nature [4].

This paper presents a simple geometric approach to analyze, a novel unsymmetrical eight bar planar parallel mechanism with three DOF. It consists of a closed chain five bar planer parallel mechanism, to which a three link serial structure is joined, forming a another closed chain mechanism. Hence, from the geometry of the mechanism, it contains three actuators, actuating each input link. In kinematic analysis - the direct kinematic analysis is done to calculate the end effector position, when the corresponding joint values are known. The direct kinematic analysis is carried out by decomposing the closed chain of the mechanism in different sub chains and selecting the origin. In the same way, the inverse kinematic analysis is done to calculate the joint values for a given end effector positions.

The method has been tested for the sample case to analyze the position of the end effector for different inputs by changing the origin of the mechanism. The analysis shows that, there is no effect on end effector position, though the origin of the mechanism gets change.

DESIGN OF THREE DOF PARALLEL LINKAGE MECHANISM

The proposed manipulator configuration consists of a closed –chain 3 DOF mechanisms acting in parallel planes providing open access for end effectors similar to serial link 3 DOF manipulator.

Drives will be located at the base. The manipulator comprises of mechanisms in two planes perpendicular to the base platform. The first plane consists of 2 DOF five bar planer manipulator with 5 links and five joints. The mechanism in the first plane is shown in Fig. 1. The second perpendicular plane on the platform consists of two binary links and the object link is connected to joint of five bar planer manipulator as shown in Fig 2.

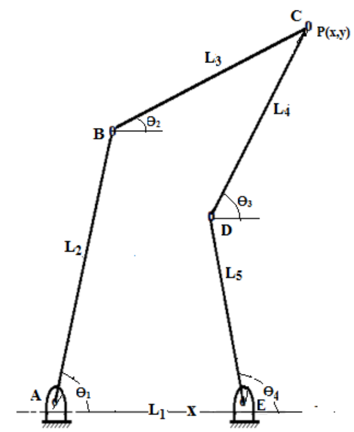

FIGURE 1.FIVE BAR PLANER MECHANISM

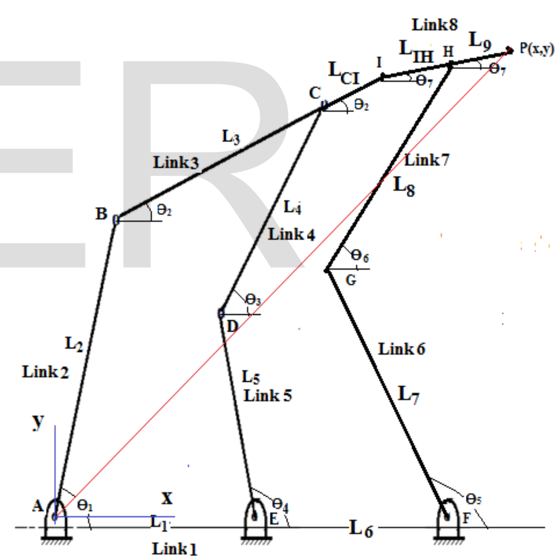

FIGURE 2.EIGHT BAR PLANER MECHANISM

The simplest form of a closed loop parallel linkage mechanism is a two degree of freedom five bar planer mechanism is shown in Fig 1. This mechanism can be used to position a point on the plane and the Cartesian coordinates associated with this manipulator are the position coordinates of one point of platform, noted (X, Y) and the actuated joint variables are Θ_1 and Θ_4. Lengths L_1, L_2, L_3, L_4 and L_5entirely define the geometry of this mechanism.

A novel unsymmetrical eight bar planar parallel mechanism with three DOF is shown in Fig 2. It consists of a closed chain five bar planer parallel mechanism, to which a three link serial structure is joined, forming a another closed chain mechanism.

Hence, from the geometry of the mechanism, it contains three actuators at **A, E** and **F**, actuating each input link.

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KINEMATIC ANALYSIS

Kinematic analysis is necessary for the mathematical analysis. The main aim is to establish the relation between joint angles and the coordinates of the end effectors. The kinematic structure of closed loop mechanism with three DOF (as shown in Fig. 2) is considered for the analysis. It consists of a Five bar planer mechanism ABCDE to which a three link serial structure FGHP is attached.Three actuators are located at A, E and F, having rotation angles denoted by Θ_1, $\Theta_4$and Θ_5 respectively.

Forward kinematics

In forward kinematics, the joint angles of three actuators, located at A, E and F, (i.e. Θ_1,$\Theta_4$and Θ_5) are known, the position of the end effector can be determined. However, in case of parallel manipulators, the forward kinematics is not so straight forwards as in serial manipulators, since their kinematics is constrained by loop closure equations. In order to simplify the analysis, the mechanism is decomposed into three Sub-chains as-

$$\overline{AP} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CI} + \overrightarrow{IH} + \overrightarrow{HP}$$

$$\overline{AP} = \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CI} + \overrightarrow{IH} + \overrightarrow{HP}$$

$$\overline{AP} = \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CI} + \overrightarrow{IH} + \overrightarrow{HP}$$

The kinematic analysis is done by selecting the origin as A, E and F

Assuming the origin of the mechanism is at A

Considering the Sub-chain 1 as shown in Fig. 3

From Fig 3, for the position of the end effector following vector equation can be derived:

$$\overline{AP} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CI} + \overrightarrow{IH} + \overrightarrow{HP} \tag{1}$$

Complex notation of the above vector equation is given by

$$\overline{AP} = L_2 e^{j\theta_1} + L_3 e^{j\theta_2} + L_{CI} e^{j\theta_2} + L_{IH} e^{j\theta_7} + L_9 e^{j\theta_7} \tag{2}$$

Therefore, Substituting the Euler's equivalent in Eq. (2) and separating real and imaginary parts, we get

$$X_P = L_2 C_1 + L_3 C_2 + L_{CI} C_2 + L_{IH} C_7 + L_9 C_7 \tag{3}$$

$$Y_P = L_2 S_1 + L_3 S_2 + L_{CI} S_2 + L_{IH} S_7 + L_9 S_7 \tag{4}$$

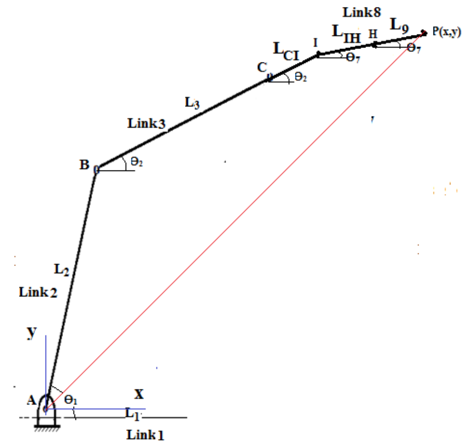


FIGURE 3.THE FIRST SUB-CHAIN

Considering the Sub-chain 2 as shown in Fig. 4

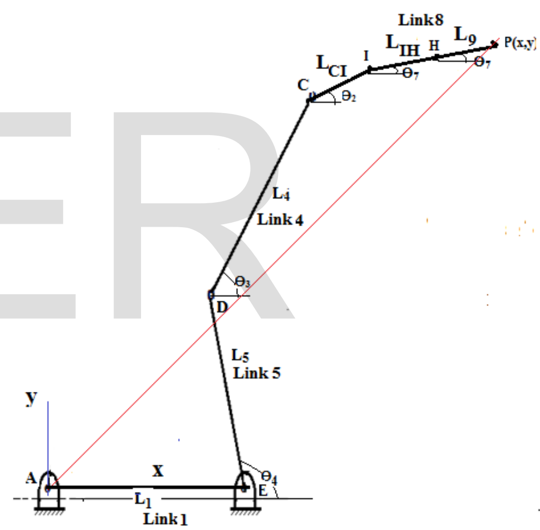


FIGURE 4.THE SECOND SUB-CHAIN

From Fig 4, for the position of the end effector following vector equation can be derived:

$$\overline{AP} = \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CI} + \overrightarrow{IH} + \overrightarrow{HP} \tag{5}$$

$$\overline{AP} = L_1 e^{j\theta} + L_5 e^{j\theta_4} + L_4 e^{j\theta_3} + L_{CI} e^{j\theta_2} + L_{IH} e^{j\theta_7} + L_9 e^{j\theta_7} \tag{6}$$

Therefore,

$$X_P = L_1 C + L_5 C_4 + L_4 C_3 + L_{CI} C_2 + L_{IH} C_7 + L_9 C_7 \tag{7}$$

$$Y_P = L_1 S + L_5 S_4 + L_4 S_3 + L_{CI} S_2 + L_{IH} S_7 + L_9 S_7 \tag{8}$$

Considering the Sub-chain 3 as shown in Fig. 5

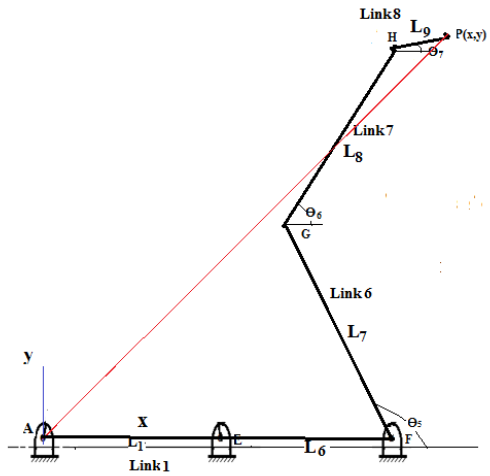


FIGURE 5.THE THIRD SUB-CHAIN

From Fig 5, for the position of the end effector following vector equation can be derived:

$$\overline{AP} = \overrightarrow{AE} + \overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GH} + \overrightarrow{HP} \tag{9}$$

$$\overline{AP} = L_1 e^{j\theta} + L_6 e^{j\theta} + + L_7 e^{j\theta_5} + L_8 e^{j\theta_6} + L_9 e^{j\theta_7} \tag{10}$$

Therefore,

$$X_P = L_1 C + L_6 C + L_7 C_5 + L_8 C_6 + L_9 C_7 \tag{11}$$

$$Y_P = L_1 S + L_6 S + L_7 S_5 + L_8 S_6 + L_9 S_7 \tag{12}$$

Equating Eq. (2) and Eq. (6)

$$L_2 e^{j\theta_1} + L_3 e^{j\theta_2} = L_1 e^{j\theta} + L_5 e^{j\theta_4} + L_4 e^{j\theta_3} \tag{13}$$

From the above complex notation, Eq. (13) the vector equation is –

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{CD} \tag{14}$$

From the geometry of the Fig. 2, the Eq. (14) indicates a Five bar mechanism, with two actuators located at A and E i.e. Θ_1 and Θ_4 respectively.

From the forward kinematics of five bar planer mechanism,

$$\theta_{3(1,2)} = 2\tan^{-1}\left(t_{3(1,2)}\right) \tag{15}$$

Where $t_{3(1,2)} = \dfrac{-F_3 \pm \sqrt{E_3^2 + F_3^2 - G_3^2}}{(G_3 - E_3)}$

$$E_3 = 2L_4(L_1 C\theta + L_5 C\theta_4 - L_2 C\theta_1)$$

$$F_3 = 2L_4(L_1 S\theta + L_5 S\theta_4 - L_2 S\theta_1)$$

$$G_3 = (L_1^2 + L_2^2 + L_4^2 + L_5^2 - L_3^2) - 2L_2L_5C(\theta_4 - \theta) - 2L_2L_5(C(\theta_4 - \theta_1) - 2L_1L_2C(\theta_1 - \theta)$$

Similarly,

$$\theta_{2(1,2)} = \tan^{-1}\left(\frac{L_4 S\theta_3 + L_5 S\theta_4 + L_1 S\theta - L_2 S\theta_1}{L_4 C\theta_3 + L_5 C\theta_4 + L_1 C\theta - L_2 C\theta_1}\right) \tag{16}$$

Equating Eq. (2) and Eq. (10)

$$L_2 e^{j\theta_1} + L_3 e^{j\theta_2} + L_{CI} e^{j\theta_2} + L_{IH} e^{j\theta_7} = L_1 e^{j\theta} + L_6 e^{j\theta} + L_2 e^{j\theta_5} + L_8 e^{j\theta_6} \tag{17}$$

From the above complex notation, Eq. (17) the vector equation is –

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CI} + \overrightarrow{IH} = \overrightarrow{AE} + \overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GH} \tag{18}$$

From the geometry of the Fig. 2, the Eq. (18) indicates a six bar mechanism, with two actuators located at A and F i.e. Θ_1 and Θ_5 respectively.

From the forward kinematics –

$$\theta_{6(1,2)} = 2\tan^{-1}\left(t_{6(1,2)}\right) \tag{19}$$

Where $t_{6(1,2)} = \dfrac{-F_6 \pm \sqrt{E_6^2 + F_6^2 - G_6^2}}{(G_6 - E_6)}$

$$E_6 = 2L_8\left[(L_1 + L_6)C\theta - L_2 C\theta_1 - (L_3 + L_{CI})C\theta_2 + L_7 C_5\right]$$

$$F_6 = 2L_8\left[(L_1 + L_6)S\theta - L_2 S\theta_1 - (L_3 + L_{CI})S\theta_2 + L_7 S_5\right]$$

$$\begin{aligned} G_6 = {} & (L_1 + L_6)^2 + L_2^{\,2} + (L_3 + L_{CI})^2 + L_7^2 + L_8^2 - L_{IH}^2 \\ & + 2L_7(L_1 + L_6)\ C(\theta_5 - \theta) - 2L_2L_7C(\theta_5 - \theta_1) \\ & - 2L_7(L_3 + L_{CI})\ C(\theta_5 - \theta_2) - 2(L_1 + L_6)\ C(\theta_1 - \theta) \\ & - 2(L_1 + L_6)(L_3 + L_{CI})\ C(\theta_2 - \theta) + 2L_2(L_3 + L_{CI})\ C(\theta_2 - \theta_1) \end{aligned}$$

Similarly,

$$\theta_{7(1,2)} = \tan^{-1}\left(\frac{(L_1 + L_6)S\theta - L_2 S\theta_1 - (L_3 + L_{CI})S\theta_2 + L_7 S\theta_5 + L_8 S\theta_6}{(L_1 + L_6)C\theta - L_2 C\theta_1 - (L_3 + L_{CI})C\theta_2 + L_7 C\theta_5 + L_8 C\theta_6}\right) \tag{20}$$

Equating Eq. (6) and Eq. (10)

$$L_5 e^{j\theta_4} + L_4 e^{j\theta_3} + L_{CI} e^{j\theta_2} + L_{IH} e^{j\theta_7} = L_6 e^{j\theta} + L_7 e^{j\theta_5} + L_8 e^{j\theta_6} \quad (21)$$

From the above complex notation, Eq. (21) the vector equation is –

$$\overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CI} + \overrightarrow{IH} = \overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GH} \quad (22)$$

From the geometry of the Fig. 2, the Eq. (22) indicates a Sevenbar mechanism, with two actuators located at E and F i.e. Θ_4 and Θ_5 respectively.
From the forward kinematics –

$$\theta_{6(1,2)} = 2\tan^{-1}\left(t_{6(1,2)}\right) \quad (23)$$

Where $t_{6(1,2)} = \dfrac{-F_6 \pm \sqrt{E_6^2 + F_6^2 - G_6^2}}{(G_6 - E_6)}$

$$E_6 = 2L_8\left(L_6C\theta - L_7C\theta_5 - L_5C\theta_4 - L_4C_3 - L_{CI}C_2\right)$$

$$F_6 = 2L_8\left(L_6S\theta - L_7S\theta_5 - L_5S\theta_4 - L_4S_3 - L_{CI}S_2\right)$$

$$\begin{aligned} G_6 = {} & L_4^{\,2} + L_5^{\,2} + L_6^{\,2} + L_7^2 + L_{CI}^2 + L_8^2 - L_{IH}^2 \\ & + 2L_6L_7\,C(\theta_5 - \theta) + 2L_5L_4C(\theta_4 - \theta_3) \\ & + 2L_5L_{CI}\,C(\theta_4 - \theta_2) + 2L_4L_{CI}\,C(\theta_3 - \theta_2) \\ & - 2L_5L_6\,C(\theta_4 - \theta) - 2L_4L_6\,C(\theta_3 - \theta) \\ & - 2L_{CI}L_6\,C(\theta_2 - \theta) - 2L_5L_7\,C(\theta_5 - \theta_4) \\ & - 2L_4L_7\,C(\theta_5 - \theta_3) - 2L_{CI}L_7\,C(\theta_5 - \theta_2) \end{aligned} \quad (24)$$

Similarly,

$$\theta_{7(1,2)} = \tan^{-1}\left(\frac{L_6S\theta + L_7S\theta_5 - L_8S\theta_6 - L_5S_4 - L_4S\theta_3 - L_{CI}S\theta_2}{L_6C\theta + L_7C\theta_5 - L_8C\theta_6 - L_5C_4 - L_4C\theta_3 - L_{CI}C\theta_2}\right) \quad (25)$$

$\therefore$ Position of end effector is given by –

$$\begin{aligned} X_p &= L_2C\theta_1 + (L_3 + L_{CI})C\theta_{2(1,2)} + (L_{IH} + L_9)C\theta_7 \\ &= L_1C\theta + L_5C\theta_4 + L_4C\theta_3 + L_{CI}C\theta_2 + (L_{IH} + L_9)C\theta_7 \\ &= (L_1 + L_6)C\theta + L_7C\theta_5 + L_8C\theta_6 + L_9C\theta_7 + (L_{IH} + L_9)C\theta_7 \end{aligned} \quad (26)$$

$$\begin{aligned} Y_p &= L_2S\theta_1 + (L_3 + L_{CI})S\theta_{2(1,2)} + (L_{IH} + L_9)S\theta_7 \\ &= L_1S\theta + L_5S\theta_4 + L_4S\theta_3 + L_{CI}S\theta_2 + (L_{IH} + L_9)S\theta_7 \\ &= (L_1 + L_6)S\theta + L_7S\theta_5 + L_8S\theta_6 + L_9S\theta_7 + (L_{IH} + L_9)S\theta_7 \end{aligned} \quad (27)$$

When solution exist in Eq. (26) and Eq. (27), one input results in four sets of end effector positions

Inverse kinematics

In inverse kinematics**,** the joint angles of three actuators, located at A, E and F, (i.e. Θ_1,Θ_4 and Θ_5) for the known position of the end effector can be determined. To solve the inverse kinematics, closed loops are often broken down to branches which are treated as serial chains.

Considering sub-chain -1

From Eq. (3) and Eq. (4)

$$(L_{IH} + L_9)C_7 = X_P - L_2C_1 - (L_3 + L_{CI})C_2 \quad (28)$$

$$(L_{IH} + L_9)S_7 = X_P - L_2S_1 - (L_3 + L_{CI})S_2 \quad (29)$$

Squaring and adding Eq. (28) and Eq. (29) for eliminating θ_7

$$\theta_{1(1,2)} = 2\tan^{-1}\left(t_{1(1,2)}\right) \quad (30)$$

$$\therefore t_{1(1,2)} = \frac{-F_1 \pm \sqrt{E_1^2 + F_1^2 - G_1^2}}{(G_1 - E_1)} \ \&$$

$$E_1 = 2L_2\left[(L_3 + L_{CI})C_2 - X_P\right]$$

$$F_1 = 2L_2\left[(L_3 + L_{CI})C_2 - Y_P\right]$$

$$\begin{aligned} G_1 = {} & (X_P{}^2 + Y_P{}^2 + L_2^2 + (L_3 + L_{CI})^2 - (L_{IH} + L_9)^2 \\ & - 2X_P(L_3 + L_{CI})C\theta_2 - 2Y_P(L_3 + L_{CI})S\theta_2 \end{aligned}$$

Similarly,

$$\theta_{7(1,2)} = \tan^{-1}\left(\frac{Y_P - L_2S\theta_1 - (L_3 + L_{CI})S\theta_2}{X_P - L_2C\theta_1 - (L_3 + L_{CI})C\theta_2}\right) \quad (31)$$

Considering sub-chain -2

From Eq. (7) and Eq. (8)

$$L_{CI}C_2 = X_P - L_1C - L_4C_3 - (L_{IH} + L_9)C_7 - L_5C_4 \quad (32)$$

$$L_{CI}S_2 = X_P - L_1S - L_4S_3 - (L_{IH} + L_9)S_7 - L_5S_4 \quad (33)$$

Squaring and adding Eq. (32) and Eq. (33) for eliminating θ_2

$$\theta_{4(1,2)} = 2\tan^{-1}\left(t_{4(1,2)}\right) \quad (34)$$

$$\therefore t_{4(1,2)} = \frac{-F_4 \pm \sqrt{E_4^2 + F_4^2 - G_4^2}}{(G_4 - E_4)} \;\&$$

$$E_4 = 2\,L_1 L_5 C\theta + 2\,L_4 L_5 C\theta_3 + 2\,L_5 (L_{IH} + L_9) C\theta_7 - 2L_5 X_P$$

$$F_4 = 2\,L_1 L_5 S\theta + 2\,L_4 L_5 S\theta_3 + 2\,L_5 (L_{IH} + L_9) S\theta_7 - 2L_5 Y_P$$

$$G_4 = (X_P{}^2 + Y_P{}^2 + L_1^2 + L_4^2 + (L_{IH} + L_9)^2 + L_5^2 - L_{CI}^2$$
$$+ 2(L_{IH} + L_9) L_4 C(\theta_7 - \theta_3) + 2(L_{IH} + L_9) L_1 C(\theta_7 - \theta)$$
$$+ 2L_1 L_4 C(\theta_3 - \theta) - 2X_P [L_1 C\theta + L_4 C\theta_3 + (L_{IH} + L_9) C_7]$$
$$- 2Y_P [L_1 S\theta + L_4 S\theta_3 + (L_{IH} + L_9) S_7]$$

Similarly,

$$\theta_{2(1,2)} = \tan^{-1}\left(\frac{Y_P - (L_{IH} + L_9) S\theta_7 + L_4 S\theta_3 + L_1 S\theta - L_5 S_4}{X_P - (L_{IH} + L_9) C\theta_7 + L_4 C\theta_3 + L_1 C\theta - L_5 C_4} \right) \quad (35)$$

Considering sub-chain -3

From Eq. (11) and Eq. (12)

$$L_8 C_6 = X_P - (L_1 + L_6) C - L_7 C_5 - L_9 C_7 \quad (36)$$

$$L_8 S_6 = Y_P - (L_1 + L_6) C - L_7 S_5 - L_9 S_7 \quad (37)$$

<u>Squaring and adding Eq. (36) and Eq. (37) for eliminating θ_6</u>

$$\theta_{5(1,2)} = 2\tan^{-1}(t_{5(1,2)}) \quad (34)$$

$$\therefore t_{5(1,2)} = \frac{-F_5 \pm \sqrt{E_5^2 + F_5^2 - G_5^2}}{(G_5 - E_5)} \;\&$$

$$E_5 = 2\,L_7 [(L_1 + L_6) C\theta + L_9 C\theta_7 - X_P]$$

$$F_5 = 2\,L_7 [(L_1 + L_6) S\theta + L_9 S\theta_7 - Y_P]$$

$$G_5 = (X_P{}^2 + Y_P{}^2) + (L_1 + L_6)^2 + L_9^2 + (L_7^2 - L_8^2)$$
$$+ 2L_9 (L_1 + L_6) C(\theta_7 - \theta)$$
$$- 2X_P [(L_1 + L_6) C\theta + L_9 C\theta_7]$$
$$- 2Y_P [(L_1 + L_6) S\theta + L_9 S\theta_7]$$

Similarly,

$$\theta_{6(1,2)} = \tan^{-1}\left(\frac{Y_P - (L_1 + L_6) S\theta - L_7 S\theta_5 + L_9 S\theta_7}{X_P - (L_1 + L_6) C\theta - L_7 C\theta_5 + L_9 C\theta_7} \right) \quad (31)$$

The similar analysis is done by taking the origin as E and then F instead of A. it is observed that the equations for θ_2, θ_3, θ_6 and θ_7 is same.

METHOD VALIDATION

The two proposed method of analysis is validated by actual calculations of forward and inverse kinematics; results obtained are compared with the graphical analysis. For the same, a mechanism with following link lengths is selected and the result obtained is as follows-

Forward kinematics

In forward kinematics, two cases are considered having different link lengths. The input joint angles are arbitrarily chosen to check the validity of the equations, for which the output angles and position of the end effector is determined.

Table 1.RESULT OF FORWARD KINEMATICS

Sn	Link lengths	Input angles			No. of sol^n	End effector position	
		Θ_1	**Θ_4**	**Θ_5**		**X_p**	**Y_p**
1	L_1 = 50, L_2 = 50 L_3 = 66, L_4= 50 L_5 = 50, L_6= 50 L_{CI}= 25, L_7= 70 L_8= 55, L_9= 25	67	96	102	1	**126.80**	**125.7**
					2	61.11	32.15
					3	28.14	109.7
					4	118.64	40.01
2	L_1 = 50, L_2 = 40 L_3 = 40, L_4= 42 L_5 = 42, L_6= 50 L_{CI}= 30, L_7= 90 L_8= 60, L_9= 30	100	70	130	1	**102.98**	**37.96**
					2	-4.58	19.18
					3	Solution does not exist, due to complex Roots of Θ_6	
					4	131.00	74.00

Hence, from the above analysis, it has been observed that, the forward kinematic has foursolutions and this result is validated by graphical method.

Inverse kinematics

In inverse kinematics, same two different cases are considered. The position of the end effector obtained from the forward kinematic solution is kept same for which the joint angles are determined.

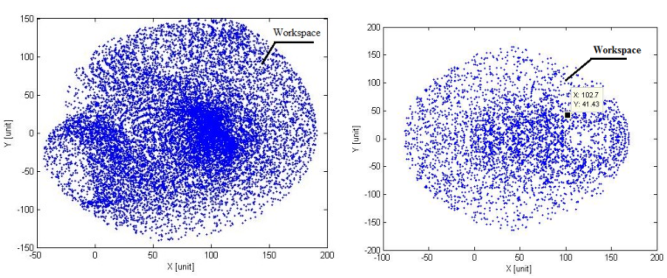
Table 2.RESULT OF INVERSE KINEMATICS

Sn	Link lengths	End effector position			No of sol	Input angles		
		X_p	Y_p	θ_7		θ_1	θ_4	θ_5
1	$L_1 = 50$ $L_2 = 50$ $L_3 = 66$ $L_4 = 50$ $L_5 = 50$ $L_6 = 50$ $L_{CI} = 25$ $L_7 = 70$ $L_8 = 55$ $L_9 = 25$	**126.80**	**125.7**	**11.12**	1	**66.76**	**95.86**	**101.9**
					2	66.76	95.86	75.92
					3	66.76	70.24	101.9
					4	66.76	70.24	75.92
					5	45.61	97.91	101.9
					6	45.61	97.91	75.92
					7	45.61	62.78	101.9
					8	45.61	62.78	75.92
2	$L_1 = 50$ $L_2 = 40$ $L_3 = 40$ $L_4 = 42$ $L_5 = 42$ $L_6 = 50$ $L_{CI} = 30$ $L_7 = 90$ $L_8 = 60$ $L_9 = 30$	**102.98**	**37.96**	34.41	1	**100.0**	**69.99**	**129.9**
					2	100.0	69.99	142.0
					3	100.0	-151.1	129.9
					4	100.0	-151.1	142.0
					5	-91.3	143.9	142.0
					6	-91.3	143.9	129.9
					7	-91.3	-70.3	142.0
					8	-91.3	-70.3	129.9

Hence, from the above analysis, it has been observed that, the inverse kinematic has eight solutions

WORKSPACE ANALYSIS

Workspace is the region, where the end effector of the robot can reach, when the input is given in terms of joint angles. In this paper the workspace is determined for the above four mentioned cases. For the analysis of the workspace, a Mat lab program is used.



(a)(b)

FIGURE 7.WORK SPACE FOR DIFFERENT CASES **(a)** CASE 1, **(b)** CASE 2

CONCLUSIONS

This paper reports kinematic analysis of complex parallel mechanisms and to identify the workspaces. Since the mechanism is complex, it is divided into subchains for solving its kinematics.The proposed method enables us to project, the multiple solutions of kinematic analysis of parallel mechanisms. In this paper, the example of eight bar planer mechanism is taken, for which the four solutions of forward kinematics and eight solutions of inverse kinematics were identified. This work can be extended further in future for identifying the singularities for parallel mechanisms.

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